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STABILITY OF WAVELENGTHS AND SPATIOTEMPORAL INTERMITTENCY IN COUPLED MAP LATTICES

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Abstract

In relation to spatiotemporal intermittency, as it can be observed in coupled map lattices, we study the stability of different wavelengths in competition.

Introducing a two dimensional map, we compare its dynamics with the one of the whole lattice. We conclude a good agreement between the two.

The reduced model also allows to introduce an order parameter which combines the diffusion parameter and the spatial wavelength under consideration.

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1. INTRODUCTION

Coupled Map Lattices (CML) have been proposed as simplest models for space-time complexity.

Spatially extended dynamical systems are typically present in many experimental situations.

Examples of such systems can be found in hydrodynamics, plasma physics, chemical reactions or biological systems. For those non-linear systems, CML gives a useful alternative point of view to the more conventional tool of partial differential equations.

The advantage of CML is based on the use of local maps with well controlled dynamical properties, that are synchronously updated and with suitably chosen local or global coupling [1-4].

Different models of CML have been widely investigated from various point of view: existence and properties of an invariant measure for infinite lattices at low coupling [5], stability and universal properties of homogeneous states [3], [7], [8], [9], study of the dynamical regime by means of a bi-orthogonal decomposition [10], [11], [17] and, probably the most important issue in view of the physical applications as they appear, for instance in Rayleigh-Bénard convection [12], the understanding of spatiotemporal intermittency [1-3] and [13-16]. The emergence of spatial coherent structures from a local chaotic dynamics in CML was discussed in [6]. On the other hand, the coexistence of different domains in the lattice with coherent dynamical behaviour, separated by boundaries (kinks), was one of the first and fascinating features observed in these models.

The relation of this situation with spatiotemporal intermittency has been mentioned by several authors [1], [4], [15] and was named "natural wavelengths" by the later authors.

In a way, this mechanism can be described as follows: the size of the different coherent regions is modified by a slow displacement of the kinks, until a destabilization of the periodic structure occurs inside them. Then, after a transient regime of variable duration, a new periodic state is restored in those regions. In fig. 1-a, b and c we present three plots of the dynamics for typical situations.

There, we can see spatiotemporal representations of a one-dimensional lattice - see (2.1) for an exact definition of the model - for different values of the parameters. The stro-

boscopic period $\Delta T = 512$ or 256 is chosen according to the time scale of the modifications observed in the lattice.

As already pointed out by Kaneko [1], [4] and Keller and Farmer [15], the slow motion of the kinks is due to the presence, between the two regions, of a nodal point x_i , lying in the neighbourhood of the fixed point of the logistic map.

In any case, the spatial wavelength is modulated by the motion of the domain walls until the destabilization of this wavelength occurs as a threshold phenomenon.

It will be shown later that bifurcations of the temporal period of the dynamics may also occur.

It is also obvious that, inside each region, the wave number, an integer, must be constant between two consecutive unstable situations. Therefore, changing the size of the region, will involve a continuous change of the spatial wavelength λ , of the corresponding configuration.

From this point of view it seems natural to study the stability of these wavelengths. They act as a bifurcation parameter in this transition between local order and chaos as was first observed in [15], and therefore we guess that it may help in the understanding of spatiotemporal intermittency.

The aim of the present work is to study analytically and numerically some features of this mechanism. Let us mention the two main ingredients used in our paper.

First, we propose a two-dimensional mapping depending on two parameters, whose dynamics (cycles, bifurcations) may describe the main features of the whole lattice. In a sense that will become clearer later, we can say that the whole CML dynamics is enslaved by the one of this simple model.

The first parameter of the reduced map is related to the nonlinearity of the local map in the CML (here the logistic map) describing the dynamics at each site of the lattice. The second parameter contains the diffusive constant governing the interaction between neighbouring sites together with the space wavelength and, therefore, is used as a control parameter.

The second main ingredient is used in numerical simulations. Instead of trying to describe the "real" situation as it is shown in fig.1, we build an "artificial" recipe where

the whole lattice is made of only one region and the stability of the space wavelength is studied by changing the number of sites in the lattice.

This is done by means of an "adiabatic compression" described in section 3 below and such that the reader can easily imagine by having a look at fig.2.

The paper is organized as follows:

In section 2, after a brief presentation of the models of interest, we use the Fourier transform of spatial structures to introduce the reduced model that is described in section 4. Obviously, this step is reminiscent of amplitude equations as they are used in weak turbulence [18].

In section 5 we present the simplest case for which the reduced model is exact. It will be clear that this case corresponds to the situation described in [6]. A way of using this reduced model out of the region of stability, by means of a bi-orthogonal decomposition was proposed in [17].

In section 6 we deal with the stability in presence of a time-period two dynamics and in section 7 of period four.

Section 8 deals with the case of homogeneous dynamics.

In section 9 we briefly compare the previous results with numerical computations of Lyapunov exponents and section 10 concludes.

2. THE MODELS AND THEIR FOURIER ANALYSIS.

In this work we consider a one-dimensional lattice of length L and a nonlinear map f of the unit interval into itself. The state of the system at time t in configuration space is given by x_i^t , $0 \leq x_i^t \leq 1$, $1 \leq i \leq L$. The new state at time $t+1$ is defined in each site by the following convex linear combination of the updated values of neighbour sites :

$$x_i^{t+1} = (1 - \varepsilon) f(x_i^t) + \frac{\varepsilon}{2} \{f(x_{i+1}^t) + f(x_{i-1}^t)\} \quad (2.1)$$

that we often will write :

$$x_i^{t+1} = f(x_i^t) + \frac{\varepsilon}{2} \Delta f(x_i^t) \quad (2.2)$$

where Δ is the discrete Laplacian operator.

Here ε is the coupling between neighbour sites, $0 \leq \varepsilon \leq 1$ and f is chosen as the logistic map :

$$f(x) = \mu x(1 - x), \quad 0 \leq \mu \leq 4 \quad (2.3)$$

We always consider periodic boundary conditions in (2.1), namely $x_{L+1} = x_1$, $x_L = x_0$.

In the following we shall see that most of the results obtained for the model (2.1) are still valid for a more general one, where the coupling between sites is given by a diffusion operator. This case can be considered as more realistic if one has in mind a limiting process to obtain a continuous configuration space.

For this model, already introduced in [15], we replace formula (2.1) by the following :

$$x_i^{t+1} = \sum_{j=-N}^N \rho_j f(x_{i+j}^t) \quad (2.4)$$

where N is the range of the interaction and the ρ_j are real positive numbers satisfying :

$$\sum_j \rho_j = 1 \quad (2.5)$$

and

$$\rho_j = \rho_{-j} \quad (2.6)$$

Clearly the model described by (2.1) is a particular case of this one.

In the numerical simulations of (2.4) we have chosen a sequence of linearly decreasing weights :

$$\rho_j = c \left(1 - \frac{|j|}{N+1} \right), \quad 0 \leq |j| \leq N+1 \quad (2.7)$$

where c is given by the normalization relation (2.5).

Finally the parameter μ of the logistic map will always verify $\mu > \mu_c$, where $\mu_c = 3,5699\dots$ is the critical value above which the map f has a chaotic dynamics.

In some cases we will have to consider values of μ such that $\mu_c < \mu < \mu_1 = 3,677\dots$ for which the dynamics of f is attracted inside two disjoint intervals Δ_1 and Δ_2 whose extreme points are determined by the four first iterated by f of the point $x = 1/2$.

Then, every point of Δ_1 (resp Δ_2) is mapped by f in a point of Δ_2 (resp Δ_1).

For $\mu > \mu_1$ this two intervals overlaps.

Let's now concentrate on the model (2.1).

Since we are interested in the study of space periodic configurations, it is natural to introduce their discrete Fourier transforms.

In absence of convective terms, we can restrict ourselves to even configurations, bringing to the following representation :

$$x_i^t = \sum_{k=0}^{L/2} a_k^t \cos \left(\frac{2\pi k i}{L} \right) \quad (2.8)$$

from where immediatly follows :

$$\Delta x_i^t = \sum_{k=0}^{L/2} 2 a_k^t \left\{ \cos \left(\frac{2\pi k}{L} \right) - 1 \right\} \cos \left(\frac{2\pi k i}{L} \right) \quad (2.9)$$

Finally, using the explicit form of the logistic map, we find the equations which give the dynamics of the Fourier coefficients.

For the saving of notations we write $a_k = a_k^t$ and $\bar{a}_k = a_k^{t+1}$.

Therefore we get :

$$\begin{aligned}\bar{a}_0 &= f(a_0) - \frac{\mu}{2} \left(\sum_{i=1}^{L/2} a_i^2 + a_{L/2}^2 \right) \\ \bar{a}_{L/2} &= (1 - 2\varepsilon) \mu \left\{ a_{L/2} (1 - 2a_0) - \frac{1}{2} \sum_{k=1}^{L/2-1} a_k a_{L/2-k} \right\}\end{aligned}$$

and for $q = 1, 2, \dots, L/2 - 1$:

$$\begin{aligned}\bar{a}_q &= \mu \alpha(q, \varepsilon) \left\{ a_q (1 - 2a_0) - \frac{1}{2} \sum_{k=1}^{q-1} a_k a_{q-k} - \right. \\ &\quad \left. - \frac{1}{2} \sum_{k=0}^q a_{k+L/2-q} a_{L/2-k} - \sum_{k=1}^{L/2-q} a_{k+q} a_k \right\}\end{aligned}\tag{2.10}$$

where

$$\alpha(q, \varepsilon) = 1 - 2 \varepsilon \sin^2 \left(\frac{\pi q}{L} \right)\tag{2.11}$$

We denote by \mathcal{F} this $(L/2 + 1)$ dimensional map. Notice that, for each q , the non linear part of \mathcal{F} is a convolution product which is proportional to the sum of the $a_i a_j$ where the indices (i, j) belong to the boundary of a rectangle in the $L/2 \times L/2$ square lattice of Fourier modes defined by the corners $(0, q)$, $(q, 0)$, $(L/2, L/2 - q)$, $(L/2 - q, L/2)$. This rectangle is reduced to one of the diagonals for each of the exceptional terms \bar{a}_0 and $\bar{a}_{L/2}$.

It is worth to notice that the coupling parameter ε enters in \mathcal{F} only by means of (2.11). Therefore, the relation (2.11) has the meaning of a rescaling between ε and the set of wavenumbers q .

In order to motivate the introduction later on of a reduced model we make now a remark about the relation of equations (2.10) with the corresponding dynamics in physical space.

The first equation of (2.10) describes the dynamics of the x_i spatial mean value since

$$a_0 = \langle x \rangle = \frac{1}{L} \sum_{i=1}^L x_i\tag{2.12}$$

On the other hand, writing

$$x_i = \langle x \rangle + \delta_i \quad (2.13)$$

we can rewrite the first equation in (2.10) as well as the dynamics of the variances δ_i . Thanks to the chosen boundary conditions we may have $\langle \Delta f(x) \rangle = 0$ and therefore we get

$$\begin{aligned} \overline{\langle x \rangle} &= f(\langle x \rangle) - \mu \langle \delta^2 \rangle \\ \bar{\delta}_i &= \mu(1 - 2\langle x \rangle) \left(1 + \frac{\varepsilon}{2}\Delta\right) \delta_i - \mu \left(1 + \frac{\varepsilon}{2}\Delta\right) (\delta_i^2 - \langle \delta^2 \rangle) \end{aligned} \quad (2.14)$$

where, again u , \bar{u} stand for the values of u at time t and $t + 1$.

Coming back to (2.10) we also remark that, if at time t_0 we start with a configuration for which the only non vanishing coefficients are a_0 and a_{q_0} ($q_0 \leq L/4$) then at time $t_0 + 1$ the only new created harmonics corresponds to a_{2q_0} . Therefore if q_0 and ε are such that

$$\alpha(2q_0, \varepsilon) = 0 \quad (2.15)$$

the only non vanishing amplitude for any time remains a_{q_0} .

In other words, in that situation,

$$x_i^t = a_0^t + a_{q_0}^t \cos \frac{2\pi q_0 i}{L} \quad (2.16)$$

is an exact solution. This is the case, for instance for

$$\lambda_0 = \frac{L}{q_0} = 6 \quad \text{and} \quad \varepsilon = 2/3 \quad (2.17)$$

We shall see that such a selection of a spatial wavelength by means of a non linear coupling between the spatial mean value and the amplitude of the oscillation is observed in much more general situations than the ones strictly obeying (2.15).

In order to get an insight into this phenomenon it is useful to look for the behaviour of $\alpha(q, \varepsilon)$ as a function of q , for a given ε . As an example this function is plotted in fig. 3 for $\varepsilon = 0,667$ and $L = 100$. The behaviour of α near to the point where $\alpha = 0$ indicates we may expect the damping of all a set of harmonics.

We can consider the case $\lambda = 2$ ($q_0 = L/2$) to be very special, since then the system evolves according to only the first two equations in (2.10) where $a_{L/2} = a_{q_0}$ is coupled only to the mean value a_0 . No condition as (2.15) is required for the existence of an exact solution given by (2.16). We will come back to this case in section 5.

To end up this section we briefly indicate how the previous results should be modified if, instead of the simplest model (2.1) we take the more general diffusive model (2.4).

The dynamics of Fourier coefficients obeys the same equations as (2.10) where we only need to replace $\alpha(q, \varepsilon)$ by :

$$\alpha(q, \{\rho\}) = 1 - 4 \sum_{j=1}^N \rho_j \sin^2 \left(\frac{\pi q j}{L} \right) \quad (2.18)$$

where N is the range of the diffusion operator. Let us notice that (2.18) is a good candidate for a limiting process to a continuous space.

Then, the equations for the dynamics of the mean value and variances, similar to (2.14), are the following :

$$\left| \begin{array}{l} \overline{\langle x \rangle} = f(\langle x \rangle) - \mu \langle \delta^2 \rangle \\ \overline{\delta_i} = \mu(1 - 2 \langle x \rangle) \mathcal{D}(\delta_i) - \mu \mathcal{D}(\delta_i^2 - \langle \delta^2 \rangle) \end{array} \right. \quad (2.19)$$

where the diffusion operator \mathcal{D} is defined as

$$\mathcal{D}(x_i) = \sum_{j=-N}^N \rho_j f(x_{j+i}) \quad (2.20)$$

and we used the normalization (2.5) and symmetry (2.6) for the ρ_j as well as the periodic boundary conditions.

Fig. 4 shows a plot of the function $\alpha(q, \{\rho\})$ where the special form of the ρ_j given by (2.7) is considered. Compared with Fig. 3, the faster decay of α as a function of the wave number q indicates a selection of larger wavelengths in this case and this is what we can observe numerically.

3. ADIABATIC COMPRESSION : A NUMERICAL RECIPE

As it was already mentioned in the introduction about the "natural" model, (i.e. the one build on a lattice of constant size L), the spatial wavelengths which are present during the dynamics are modulated by the motion of the their respective domain walls. Our aim is to study the instabilities which give rise to these spatiotemporal oscillations as the source of spatiotemporal intermittency.

In order to compare this situation in the lattice with the behaviour of the simplified model presented in Section 4 we propose to isolate that phenomenon by means of a numerical experiment. We call it adiabatic compression of the lattice that we will describe now.

When the lattice of size L reaches a periodic state (eventually after a transient regime) we carry out the following transformation :

- (i) L becomes $L' = L - 1$
- (ii) x_i becomes $x'_i = x_i + (i - 1)(x_{i+1} - x_i)/(L' - 1)$
- (iii) the periodic boundary conditions are restored for the new lattice. (3.1)

Notice that this transformation preserves the wavenumber but may change the corresponding wavelength.

After relaxation we may observe if the new wavelength of spatial oscillations is stable or not.

We repeat the operation until we notice a transition to a disordered state. Then the critical value λ_{cr} is given by $\lambda_{cr} = L_1/q$ where q is the wavenumber and L_1 the actual length of the lattice.

After this transition the lattice will relax (with constant length L_1) to a new periodic state with new wavenumber q_1 and, therefore, a wavelength $\lambda_1 = L_1/q_1$. The same process may be repeated now.

Coming back to fig. 2.a we can see there, two adiabatic compressions followed by the correspondent instability transitions.

In this example the model is as (2.1) with $\mu = 3.63$ and $\varepsilon = 0.667$. The initial number of sites in the lattice is $L = 85$ and the wavenumber is $q = 10$ ($\lambda = 8.5$). After adiabatic compression the first destabilization occurs for $L_1 = 82$ and then $\lambda_{cr} = 8.2$.

After relaxation the new wavenumber is $q_1 = 7$ and therefore $\lambda = 82/7 = 11.7$.

After a new adiabatic compression the system undergoes another destabilization for $L_2 = 57$ and therefore $\lambda'_{cr} = 57/7 = 8.1$ which now relaxes to a new wavenumber $q_2 = 5$ which corresponds to $\lambda' = 11.4$.

Fig. 2.b gives more details on the first described adiabatic compression, since we use now a stroboscopic period $\Delta T = 32$ instead of the $\Delta T = 256$ used in fig. 2.a. We can see how the initial perturbation propagates along the lattice and then relaxes to the new laminar state with $q_1 = 7$.

One may observe that both destabilizations are obtained for close values of λ ($\lambda_{cr} = 8.2$ and $\lambda'_{cr} = 8.1$) as well as are close the two values of the spatial wavelengths selected after relaxation ($\lambda = 11.7$ and $\lambda' = 11.4$).

Notice that $(\lambda_{cr} - \lambda'_{cr})$ as well as $(\lambda - \lambda')$ are as small as they are allowed to by the rational approximation determined by the maximum size of the lattice in this case and the correspondent integer wavenumbers. So we may wonder if by taking a much larger lattice at the beginning a better agreement of these values may be found. This is effectively true, but the price to pay is an extremely larger relaxation time (transients) and so even for a small gain of precision, we refrain to pursue in this direction.

Instead we may notice the time period of the dynamics represented in fig. 2 also undergoes a bifurcation above $\lambda = 11$, a fact that we will address later on. The reason why this is not observed in fig. 2 is simply because there, the stroboscopic periods are always multiples of the time periods of the dynamics.

We will use several times the adiabatic compression to test the predictions of the simplified model described in the following section, model where λ is taken as the varying control parameter.

4. A REDUCED MODEL

We define the following two-dimensional dynamical system :

$$\begin{cases} X_{t+1} &= f(X_t) - \frac{\mu}{2} Y_t^2 \\ Y_{t+1} &= \alpha(\lambda, \varepsilon) f'(X_t) Y_t \end{cases} \quad (4.1)$$

where $\alpha(\lambda, \varepsilon) = 1 - 2\varepsilon \sin^2(\pi/\lambda)$ and f is defined as the logistic map with parameter μ as before. Let's denote F the map defined by (4.1). Remark that now α is considered as a function of continuous parameters λ and ε .

The definition (4.1) comes from a matching of equations (2.10) describing the dynamics of the Fourier modes and the corresponding formulation (2.14) in terms of mean and variances.

It can also be deduced from (2.10) by means of the first terms of an expansion for small amplitudes compared with the mean value, in which case X_t represents the space mean value of the configuration at time t and Y_t the global amplitude of the variance at the same time.

Even if the real meaning of Y_t is to be taken with caution, we got a good agreement of the values of Y_t according the map (4.1) and the variance of the configuration at same time as it can be computed from direct numerical simulation with the whole lattice.

The only exception is for $\lambda = 2$ where the factor $\mu/2$ in the first equation of (4.1) must be replaced by μ if we want to maintain this interpretation for Y . This is due to the periodic boundary conditions. Instead of changing the form of the map F for this special case $\lambda = 2$, we prefer to keep it in the same form as (4.1) since this simply corresponds to a rescaling $Y' = Y/\sqrt{2}$ which obviously do not change the properties of (4.1).

Any way, we take (4.1) as a starting point and we will show how it behaves on the parameter space. Different bifurcations and dynamical regimes of (4.1) will be interpreted and then compared to the corresponding ones for the whole lattice.

Notice the map F depends on three parameters : μ , ε and λ , but the later two enter only in (4.1) by means of the function $\alpha(\lambda, \varepsilon)$. Therefore F really depends only on two parameters : μ and α .

We shall compute different cycles of F (for different temporal periods T) and determine their stability domains in parameter space.

When expressed as a function of ε the result will be compared to numerical simulations showing bifurcation diagrams of the whole CML as a function of ε .

On the other hand, as already written, when the results are expressed as a function of λ , they will be confirmed by applying to the CML the adiabatic compression defined in Section 3.

Before going in the more detailed study of the various dynamical regimes of the map F , we notice a special feature of it that helps to understand some of the results described below.

The second equation in (4.1) only fixes the ratio Y_{t+1}/Y_t and therefore is invariant by scaling of the variable Y as far as the condition $0 \leq Y \leq 1$ is fulfilled. This allows the translation described by the first equation to stabilize the logistic map governing the dynamics of X .

Then, the coupling between the two variables is closed because, in order that Y_t stays in a bounded domain, this stable orbit for X should be such that $\alpha f'(X_t)$ oscillates around 1 (the case where it is always, in modulus, strictly less than 1 is addressed in Section 8).

Therefore, at the end we see that the dynamics is governed by a balance of the size of the oscillations of the mean variable X and the one of the variance amplitude that may stabilize each other.

5. THE SIMPLEST CASE.

We begin our analysis with the simplest dynamics of map (4.1), namely fixed points ($T = 1$).

The coordinates of such a fixed point obey the following system of equations :

$$\begin{cases} X &= f(X) - \frac{\mu}{2}Y^2 \\ Y &= \alpha f'(X) \cdot Y \end{cases} \quad (5.1)$$

From the second equation of (5.1) we can see that an obvious fixed point holds with $Y = 0$. But then we discover from the first equation that it corresponds to the (unstable) fixed point of the logistic map, $X = f(X)$. According to the previous section, this should be an homogeneous state (with vanishing variance) and it is easy to show it is not stable. In section 8, we will come back to this issue.

Non obvious solutions ($Y \neq 0$) are easily computed as :

$$\begin{cases} X &= \frac{1}{2} \left(1 - \frac{1}{\alpha\mu} \right) \\ Y &= \left\{ \frac{1}{2\mu} \left(\mu - 2 + \frac{1}{\alpha} \right) \left(1 - \frac{1}{\alpha\mu} \right) \right\}^{1/2} \end{cases} \quad (5.2)$$

Here $\alpha = \alpha(\lambda, \varepsilon) = 1 - 2\varepsilon \sin^2\left(\frac{\pi}{\lambda}\right)$. Notice we are only interested in fixed points for which $0 \leq X \leq 1$ and $Y^2 \leq 1$. These conditions fix the boundaries, in parameter space, for the existence of solutions with $T = 1$.

The conditions for the stability of such fixed points are found by computing the eigenvalues of the tangent map of (4.1) evaluated on the solutions given by (5.2) :

$$Q_1 = \begin{vmatrix} 1/\alpha & -\mu Y \\ -2\alpha\mu Y & 1 \end{vmatrix} \quad (5.3)$$

As it can be easily shown, for the values of μ that we are interested in and unless $\lambda = 2$ or $\lambda = 3$, the stability of these fixed points will imply $\varepsilon > 1$.

We have numerically confirmed the predictions also in this case, but we refrain to present them here since they go beyond the frame of the present work, see [19].

We then focus on the simplest case, $\lambda = 2$, in order to show we can recover the results shown in [6].

First we observe that :

$$\alpha(2, \varepsilon) = 1 - 2\varepsilon \quad (5.4)$$

and by changing variables :

$$\begin{cases} A &= X + \frac{Y}{\sqrt{2}} \\ B &= X - \frac{Y}{\sqrt{2}} \end{cases} \quad (5.5)$$

the equations of the fixed points of (4.1) read :

$$\begin{cases} A &= (1 - \varepsilon) f(A) + \varepsilon f(B) \\ B &= (1 - \varepsilon) f(B) + \varepsilon f(A) \end{cases} \quad (5.6)$$

so that (5.1) reduces to a well known form. A and B are the values of the configuration for even and odd sites on the lattice and, therefore, the existence of a solution for (5.6) is equivalent to the one of the given type for the whole lattice.

In other words, as already mentioned in section 3, the reduced map (4.1) is exact when $\lambda = 2$.

From (5.6) we easily find that A and B are the roots of the equation :

$$u^2 - Su + P = 0 \quad (5.7)$$

where

$$S = A + B = 1 - \frac{1}{\mu(1 - 2\varepsilon)} \quad (5.8)$$

and

$$P = AB = \frac{-\varepsilon}{\mu(1 - 2\varepsilon)} \left(1 - \frac{1}{\mu(1 - 2\varepsilon)} \right) \quad (5.9)$$

which obviously agree with (5.1) where α is taken as in (5.4).

On the other hand, the stability of such fixed points is given by the condition

$$|\nu_{\pm}| < 1 \quad (5.10)$$

where ν_{\pm} and the two roots of the usual equation deduced from the jacobian of (5.6) namely :

$$\nu^2 - \mathbf{T}\nu + \mathbf{D} = 0 \quad (5.11)$$

where :

$$\begin{cases} \mathbf{T} &= (1 - \varepsilon) \{f'(A) + f'(B)\} \\ \mathbf{D} &= (1 - 2\varepsilon) \{f'(A) \cdot f'(B)\} \end{cases} \quad (5.12)$$

with $f'(x) = \mu(1 - 2x)$.

In fig. 5 we show the diagram of bifurcations in parameter space (μ, ε) , where the region corresponding to stable fixed points is filled in grey.

We also mention it is possible to show that the stability of the solution of (5.6) implies the stability of the corresponding solution for the whole lattice (the proof is given in the Appendix at the end of the paper).

For that purpose, we notice that the eigenvalue equations for the tangent map of the CML with a lattice of even length, calculated at the fixed point defined above may be reduced, by symmetry, to a set of $L/2$ eigenvalue problems for 2×2 matrices. Finally, inspecting the different cases, we found that, inside the stability domain of the reduced system all eigenvalues of the CML tangent map are, in modulus, strictly less than 1. Furthermore, the two bifurcations of (5.6) are of different nature. For the lower end point of the stability interval the eigenvalues are real and they are complex for the upper end point.

Let us add that, in principle, it is possible to follow the same lines of equations (5.6) for solutions of the CML with $T = 1$ and $\lambda > 2$. This leads to a set of λ -equations replacing (5.6) that was studied in [19], but the system became rapidly cumbersome and it can only be studied by numerical computation.

The advantage of our reduced model (4.1) comes from the fact that we always have a two-dimensional system, no matter of the wavelength we consider.

6. TIME-PERIOD TWO DYNAMICS

We come back again to the reduced model F defined in (4.1) to analyse the case of solutions with time period 2.

Numerical simulation shows the existence of period 2 cycles satisfying the following symmetry relation :

$$\begin{cases} X_{t+1} &= X_t \\ Y_{t+1} &= -Y_t \end{cases} \quad (6.1)$$

No other cycle of period 2 was found for F .

The coordinates X , Y of such cycles obey then the equations :

$$\begin{cases} X &= f(X) - \frac{\mu}{2}Y^2 \\ Y &= -\alpha f'(X) \cdot Y \end{cases} \quad (6.2)$$

where, again, the dependance of $\alpha = 1 - 2\varepsilon \sin^2(\frac{\pi}{\lambda})$ in λ and ε has been omitted.

The solution of (6.2) is immediately found :

$$\begin{cases} X &= \frac{1}{2} \left(1 + \frac{1}{\alpha\mu} \right) \\ Y &= \left\{ \frac{1}{2\mu} \left(\mu - 2 - \frac{1}{\alpha} \right) \left(1 + \frac{1}{\alpha\mu} \right) \right\}^{1/2} \end{cases} \quad (6.3)$$

The corresponding tangent map is, according to (4.1) given by :

$$DF(X, Y) = \begin{bmatrix} f'(X) & -\mu Y \\ \alpha f''(X) \cdot Y & \alpha f'(X) \end{bmatrix} \quad (6.4)$$

and therefore, for $Q_T = Q_2 = DF(X, -Y) \cdot DF(X, Y)$, one has :

$$Q_2 = \begin{bmatrix} \frac{1}{\alpha^2} - 2\alpha\mu^2 Y^2 & -\mu \left(1 - \frac{1}{\alpha} \right) Y \\ -2\mu(1 - \alpha)Y & 1 - 2\alpha\mu^2 Y^2 \end{bmatrix} \quad (6.5)$$

and then the determinant and trace of Q_T are :

$$\begin{cases} \det Q_2 &= \left(\frac{1}{\alpha} - 2\alpha\mu^2 Y^2 \right)^2 \\ T_r Q_2 &= 1 + \frac{1}{\alpha^2} - 4\alpha\mu^2 Y^2 \end{cases} \quad (6.6)$$

from which we can compute the eigenvalues ν_{\pm} as roots of the corresponding usual equation.

As an example of the result accuracy comparing to the behaviour of the CML, we first show a case where $\mu = 3.8$ and $\lambda = 6$ are kepted fix and ε varies. For this case fig. 6-a shows the largest value of the modulus of the eigenvalues of Q_2 as a function of ε . This function is equal to 1 for $\varepsilon_1 = 0.39\dots$ in the domain where the eigenvalues are complex and for $\varepsilon_2 = 0.695\dots$ where they are real.

Fig. 6-b shows the bifurcation diagram of a CML of 60 sites for which the two corresponding values of bifurcation are $\varepsilon'_1 = 0.4\dots$ and $\varepsilon'_2 = 0.684$. As explained in the introduction larger lattice gives an even better agreement.

Fig. 6-c gives, for the same interval in ε , the bifurcation diagram of the second coordinate $-Y-$ of the two-dimensional map F , where we can see the relation between F and the dynamics of the CML is deeper than the only coincidence of the two bifurcation points. This relation will be treated elsewhere using the method described in [17].

As a second test we take $\varepsilon = 0.667$ to be fixed as well as $\mu = 3.8$, the same value as before and taking now λ as order parameter.

The largest value of the modulus of the eigenvalues of Q_2 is now plotted as a function of λ in fig. 7.

We found $\lambda_1 = 5.96$ and $\lambda_2 = 7.98$ for the corresponding values of bifurcation. Notice that, due to the form of the function $\alpha(\lambda, \varepsilon)$, λ_1 corresponds to ε_2 and λ_2 to ε_1 .

Following the discussion of section 3 we compare now these values with the actual CML bifurcation diagram by using an adiabatic compression.

We begin with a snapshot of a lattice which length is $L = 63$ and a wavenumber $q = 9$ ($\lambda = 7$) as shown in fig. 8-a. Fig. 8-b shows the lattice after adiabatic compression until $L = 55$, where we can see the relaxed state with the same wavenumber is still stable. The

destabilization occurs for $L = 54$ ($\lambda'_1 = 6$) as it is seen in fig. 8-c. Then the system, relaxes always with $L = 54$, to a new stable state as it is shown in fig. 8-d after a transient regime of about 4000 iterations. Fig. 8-e shows a superposition of two such states at time t and $t + 2$. Notice that in fig. 8-d and 8-e we are concerned with states of temporal period 4 and a more complex spatial behaviour which is reminiscent of the Eckhaus instability in Rayleigh-Bénard convection [18].

Finally, with reference to fig. 8-f and 8-g, applying again the adiabatic compression a state with $T = 2$ is restored for $L = 48$ ($\lambda'_2 = 8$) showing again a good agreement with the prediction of the reduced map F .

The regions of parameter space where the map F has stable periodic ($T = 2$) dynamics are shown in fig. 9 in the plane (μ, ε) for different values of λ ($\lambda = 2, 4, 6, 8$) and in fig. 10 in the plane (λ, ε) for $\mu = 3.8$.

We have made a large number of numerical simulations for the CML and found they were also in good agreement with the predictions of F as they are shown in these two pictures.

We end up this section by adding that, as already noticed, changing ε in $(1 - \varepsilon)$ in equations (5.4) transforms a fixed point in a cycle of period 2. We notice that this is a special case of a more general transformation that takes equation (5.1) of a fixed point of F in equation (6.2) of a cycle of period 2, by changing α in $-\alpha$.

7. TIME-PERIOD FOUR DYNAMICS

Exactly in the same way of section 6 we can study the solutions with period 4 of F and then to compare the dynamics of F with CML one.

Again we observed that the cycles have the symmetry $Y \rightarrow -Y$ of the map F , so we only need to describe the cycle four positive numbers X_1, X_2, Y_1, Y_2 , cycle that is runned in the following way :

$$(X_1, Y_1) \rightarrow (X_2, -Y_2) \rightarrow (X_1, -Y_1) \rightarrow (X_2, Y_2) \rightarrow (X_1, Y_1) \quad (7.1)$$

This combines the permutation $X_1 \rightarrow X_2$ between Δ_1 and Δ_2 for the logistic map described in section 1 and the flip of period $T = 2$ studied before.

Therefore a cycle $T = 4$ is solution of

$$F^2(X, Y) = F(X, -Y) \quad (7.2)$$

or, in other words, solution of the system :

$$\begin{cases} X = f\left(f(X) - \frac{\mu}{2}Y^2\right) - \frac{\mu}{2}(\alpha \cdot f'(X) \cdot Y)^2 \\ \alpha^2 \cdot f'\left(f(X) - \frac{\mu}{2}Y^2\right) \cdot f'(X) = -1 \end{cases} \quad (7.3)$$

This leads, after straightforward calculations, to the following equation, expressed in the variable $Z = \alpha \cdot f'(X)$:

$$Z^2(Z^4 + \alpha^2\mu(2 - \mu)(Z^2 - 1) + 4\alpha Z) - 1 = 0 \quad (7.4)$$

Denoting Z_1, Z_2 the two real roots of (7.4) we finally get :

$$\begin{cases} X_{1,2} &= \frac{1}{2}\left(1 - \frac{Z_{1,2}}{\alpha\mu}\right) \\ Y_{1,2} &= \left\{\frac{2}{\mu}f'(X_{1,2}) - \frac{1}{\alpha^2\mu^2 f'(X_{1,2})} - \frac{1}{\mu}\right\}^{1/2} \end{cases} \quad (7.5)$$

We observe that, taking $Z = \pm 1$ we recover the equivalent expressions for $T = 1$ and $T = 2$, which is naturally related with the symmetry of the cycles and the definition of Z as the ratio of two successive values of Y .

The stability of such cycles is studied as before.

As in the previous section we only give two examples of application since all the other cases we have performed show the same accuracy.

In both cases we take $\mu = 3.63$, a value for which the attractor of f lies inside two disjoint intervals Δ_1, Δ_2 as mentioned before.

First we take $\lambda = 8$ fixed and ε as a parameter. The stability interval of the cycle of F , as it is shown in fig. 11-a, has limit points $\varepsilon_1 = 0.42$ and $\varepsilon = 0.695$ whereas the values obtained from the bifurcation diagram of the CML, shown in fig. 11-b, gave $\varepsilon'_1 = 0.41$ and $\varepsilon'_2 = 0.69$.

Then, as in section 6, we fix $\varepsilon = 0.667$ and use λ as a control parameter. From the stability of F we get stability between $\lambda_1 = 7.85$ and $\lambda_2 = 10.15$ as it is shown in fig. 12, whereas the adiabatic compression shown in fig. 13 give, for the critical values of the wavelengths, $\lambda'_1 = 65/8 = 8.1$ and $\lambda'_2 = 52/5 = 10.4$.

The same comments as in case $T = 2$, can be done for $T = 4$.

We also notice that, when the eigenvalues of Q_T cross the unit circle with complex values, the whole lattice undergoes a period doubling bifurcation.

Finally we want to add that we performed the corresponding numerical simulations for the values of the parameter for which the reduced map F has cycles of higher order and we also find a good agreement with the corresponding dynamics of the CML.

8. HOMOGENEOUS DYNAMICS

A simple observation about the dynamics of the reduced map F is worth to be noticed.

Since

$$\sup_X |f'(X)| = \mu \quad (8.1)$$

we see that the condition

$$|\alpha| \cdot \mu < 1 \quad (8.2)$$

implies, using the second equation of F that $Y_t \rightarrow 0$ when $t \rightarrow \infty$ for all initial conditions (X_0, Y_0) .

Therefore, in this case, the asymptotic dynamics is defined as

$$\begin{cases} X_{t+1} &= f(X_t) \\ Y_t &= 0 \end{cases} \quad (8.3)$$

According to the given interpretation of these variables, such a dynamics may correspond to a dynamics of the CML for which there is synchronized motion (since $Y = 0$).

The corresponding dynamics is obviously given by the one of the logistic map, according to the first equation.

This regime is already observed in CML, see [20], but suprisingly, the reduced model again gives quite accurate predictions in this case. We will address this question elsewhere.

Before ending this section, let us add the following comment : condition (8.2) is fulfilled if

$$\frac{1 + \mu}{2\mu} > \varepsilon \sin^2 \left(\frac{\pi}{\lambda} \right) > \frac{\mu - 1}{2\mu} \quad (8.4)$$

which means that, for all $\varepsilon > 0$ there is always a $\lambda(\varepsilon)$ such as for $\lambda > \lambda(\varepsilon)$ (8.4) may not be true. But this effect can not be seen if the size of the lattice is not large enough and, moreover, only for an infinite lattice we should have no such cutoff. At least in the view of the reduced model, we see some care need to be taken when dealing with finite size lattices instead of the infinite limit case.

9. LYAPUNOV EXPONENTS

Finally, let us present some results obtained by computing maximal Lyapunov exponents along orbits initialized in the vicinity of the spatiotemporal periodic states which have been studied above.

The aim is to give a more precise meaning for the terms “chaotic” and “laminar” used for a phase of a CML, here simply defined by its positive or negative Lyapunov exponents.

Again we compare the results obtained from the dynamics of the reduced model F with the ones coming from \mathcal{F} together with the explicit formulae of his tangent map.

Fig. 14-a and b shows a particular typical case for F and \mathcal{F} . Here $\mu = 3.8$ and $\lambda = 6$ whereas ε is taken as control parameter. This is the situation described in section 6, where $T = 2$.

We may observe, again, a precise connection between the two stability intervals. Moreover, for a smaller interval (0.33., 0.40..) we see that the Hopf bifurcation of the map F corresponds to the period doubling of \mathcal{F} that we have already noticed.

Many other numerical computations we have performed, for different values of the parameters, confirm this agreement between the two dynamics.

10. CONCLUDING REMARKS

In this work we were concerned with the problem of spatiotemporal intermittency as it can be observed in CML. Directed by the idea that one of the mechanisms undersetting this phenomenon is a competition of local configurations with different wavelengths we have addressed the question of the stability of such wavelengths.

The essential point of our work came from the possibility of describing many features of this bifurcation diagram as well as of the corresponding dynamics of the CML by means a two-dimensional map.

The study of this map allows, in a very simple way, to predict the existence and stability of large ordered subsets of the lattice.

This approach propounds a particular coupling between the spatial mean value and the variance of the variables in phase space. It also suggests a particular form of a control parameter that combines the diffusion constant and the spatial wavelength of the configuration.

Even if this reduced two-dimensional model gives very accurate predictions for the CML dynamics we were not able, in the general case, to achieve a complete proof of the correspondance, as it is the case for $\lambda = 2$.

Further progress will lead to the problem of the study of the full system \mathcal{F} as it was described in section 2 and would give a more serious basis to the knowledge of the conditions of applicability of the two-dimensional map F .

APPENDIX :

We show there the stability of state (5.5) of the whole lattice in the stability domain of the variable ε of the corresponding state of the reduced map (5.1).

Due to the fact that the tangent map $D\mathcal{F}$ of the whole lattice evaluated on the state (5.5) commutes with the operator of cyclic permutations of order two, whose eigenvalues are the roots of unity : $\mathcal{W}_k = e^{\frac{4\pi ki}{L}}$, $k = 1, 2, \dots, L/2$; the eigenvalues problem for $D\mathcal{F}$ may be easily reduced to the one of the set of $L/2$ matrices M_k defined by :

$$M_k = \begin{bmatrix} (1 - \varepsilon)f'(A) & \varepsilon f'(B) \left(1 + \frac{1}{\mathcal{W}_k}\right) \\ \varepsilon f'(A)(1 + \mathcal{W}_k) & (1 - \varepsilon)f'(B) \end{bmatrix} \quad (A.1)$$

We remark that $M_{L/2}$ is equivalent to the Jacobian of the reduced map whose trace \mathbf{T} and determinant \mathbf{D} are given by (5.12), and the traces \mathbf{T}_k and determinants \mathbf{D}_k of M_k may be expressed in terms of \mathbf{T} and \mathbf{D} :

$$\mathbf{T}_k = \mathbf{T} = \frac{2(1 - \varepsilon)}{1 - 2\varepsilon} ; \quad \mathbf{D}_k = \beta_k \cdot \mathbf{D} \quad (A, 2)$$

where :

$$\beta_k = 1 + \frac{\varepsilon^2}{1 - 2\varepsilon} \cdot \sin^2 \left(\frac{2\pi k}{L} \right) \quad (A, 3)$$

As shown in fig. 5, for $\mu > 3.45$, the stability domain of the reduced map extends from $\varepsilon > 0.8$. For such values the trace \mathbf{T} is always negative and β_k satisfy for any k in $1, \dots, L/2$, the relations :

$$|\beta_k| \leq 1 \quad (A, 4)$$

$$-\beta_k \leq \gamma < 1 \quad (A, 5)$$

where : $\gamma = -\beta_{L/4} = -\frac{(1-\varepsilon)^2}{1-2\varepsilon}$.

Now, with the following notations :

$$|\nu| = \max(|\nu^+|, |\nu^-|),$$

$$|\nu_\star| = \max(\max_k |\nu_k^+|, \max_k |\nu_k^-|),$$

where ν_k^\pm (resp. ν^\pm) are the eigenvalues of matrices M_k (resp. $M_{L/2}$).
and : $\tau = -\mathbf{T}$, $\Delta = \tau^2 - 4\mathbf{D}$, $\Delta_k = \tau^2 - 4\beta_k\mathbf{D}$

We are going to consider successively the cases :

1./ $D \leq 0$ and **2./** $D > 0$ with subcases

2.1/ $\Delta \geq 0$ and **2.2/** $\Delta < 0$, and show in each of them that, under the hypothesis $|\nu| < 1$, one finds : $|\nu_\star| < 1$.

1./ If $\mathbf{D} \leq 0$ then : $|\nu| = \frac{\tau + \sqrt{\tau^2 + 4|\mathbf{D}|}}{2}$ and, from : $\Delta_k = \Delta + 4(\beta_k - 1)|\mathbf{D}|$ and (A,4) it follows : $\Delta_k \leq \Delta$ for any $k = 1, 2, \dots, L/2$.

Let us denote by K_1 (resp. K_2) the subset of the indices k where $\Delta_k \geq 0$ (resp. $\Delta_k < 0$) and consider separately K_1 and K_2 :

In the first case : $|\nu_\star^{K_1}| = \max_{k \in K_1} \left(\frac{\tau + \sqrt{\Delta_k}}{2} \right) \leq |\nu|$.

In the second, where the eigenvalues are complex, one has : $|\nu_\star^{K_2}| = \max_{k \in K_2} (\sqrt{\beta_k \mathbf{D}}) \leq \sqrt{|\mathbf{D}|} < |\nu|$.

2./ If $\mathbf{D} > 0$ let us consider separately the cases of real and complex eigenvalues ν :

2.1/ if $\Delta = \tau^2 - 4\mathbf{D} \geq 0$, by use of $\Delta_k = \Delta + 4\mathbf{D}(1 - \beta_k)$ and (A.4) one finds : $\Delta_k \geq 0$ and, consequently : $|\nu_\star| = \max_k \left(\frac{\tau + \sqrt{\tau^2 - 4\beta_k \mathbf{D}}}{2} \right)$, then, with (A,5) and the relation $\mathbf{D} < \frac{\tau^2}{4}$ one obtains for $|\nu_\star|$:

$$|\nu_\star| \leq \tau \left(\frac{1 + \sqrt{1 + \gamma}}{2} \right) \text{ and, with explicit form of } \tau \text{ and } \gamma \text{ as functions of } \varepsilon : |\nu_\star| \leq \frac{1 - \varepsilon}{2\varepsilon - 1} \left(1 + \frac{\varepsilon}{\sqrt{2\varepsilon - 1}} \right) < 1.$$

2.2/ for $\Delta < 0$ one has $|\nu| = \sqrt{\mathbf{D}} < 1$, we again consider separately the subset K_3 and K_4 of indices k for which $\Delta_k \geq 0$ and $\Delta_k < 0$.

2.2.1/ if $\Delta_k \geq 0$, using again (A.5) : $|\nu_\star^{K_3}| = \max_{k \in K_3} \left(\frac{\tau + \sqrt{\tau^2 - 4\beta_k \mathbf{D}}}{2} \right) \leq \frac{\tau + \sqrt{\tau^2 + 4\gamma}}{2}$ and, as a function of ε : $|\nu_\star^{K_3}| \leq \frac{1 - \varepsilon}{2\varepsilon - 1} (1 + \sqrt{2\varepsilon}) < 1$.

2.2.2/ for k such that $\Delta_k < 0$ ($k \in K_4$) one has : $|\nu_\star^{K_4}| = \max_{k \in K_4} (\sqrt{\beta_k \mathbf{D}}) \leq \sqrt{\mathbf{D}} = |\nu|$ which ends the proof.

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FIGURES CAPTIONS

Fig. 1 : Three dimensional views of spatiotemporal evolution of a CML of 100 sites (x axis) obtained by 100 successive plots (t axis). Half of the lattice has been initialized in Δ_1 (resp. Δ_2) with $\varepsilon = 0.667$. a) $\mu = 3.6$ and the plotting time period is $\mathcal{T} = 512$. b) $\mu = 3.62$ and $\mathcal{T} = 512$. c) $\mu = 3.63$ and $\mathcal{T} = 256$.

Fig. 2 : Three dimensional views of adiabatic compressions of a CML with initial length $L = 85$, wavenumber $q = 10$ ($\lambda = 8.5$) and initial state : $x_i^0 = 0.45 + 0.15 \sin(2\pi qi/L)$. $\varepsilon = 0.667$; $\mu = 3.63$. a) Among 120 successive plots (period $\mathcal{T} = 256$) two destabilizations occur for $L = 82$ and $L = 57$. b) The same as the beginning of a), the destabilization observed for $L = 82$ is shown in much more details thanks to the lower plotting period $\mathcal{T} = 32$.

Fig. 3 : A continuous plot of the function $\alpha(q, \varepsilon)$ defined in (2.11) versus q for $\varepsilon = 0.667$ and $L = 100$.

Fig. 4 : A continuous plot of the function $\alpha(q, \{\rho\})$ defined in (2.18) versus q for $L = 100$ and linearly decreasing weights ρ of range 5.

Fig. 5 : Stability domain (grey area) of the fixed point of map F in parameter space $\varepsilon \times \mu$. (ε : horizontal axis; μ : vertical axis).

Fig. 6 : Stability of period two cycles versus ε with $\lambda = 6$ and $\mu = 3.8$. a) Plot of the largest value of the modulus of the eigenvalues of Q_2 given in (6.5) versus ε . b) Bifurcation diagram of a CML of length $L = 60$ (Horizontal axis : ε) initialized for each ε as : $x_i^0 = B + D \cos(2\pi i/L)$ where B and D are taken in the vicinity of period two cycle (6.3) of the reduced map F . For each ε a transient of $t = 200$ has been died and 100 iterations of the CML are drawn with a plotting period $\mathcal{T} = 2$. c) Bifurcation diagram of reduced map F . For each ε , Y coordinate of F is plotted for 200 iterations after a transient of 200 has been died.

Fig. 7 : Plot of the largest value of the modulus of eigenvalues of Q_2 versus λ with $\mu = 3.8$ and $\varepsilon = 0.667$.

Fig. 8 : A sequence of transitions between period two states of a CML submitted to adiabatic compressions, with $\varepsilon = 0.667$, $\mu = 3.8$ and a stroboscopic period $\mathcal{T} = 2$. a) Initial state of a CML of length $L = 63$ and wavenumber $q = 9$ ($\lambda = 7$). b) After successive compressions $L = 55$. The lattice is still stable but a modulation of spatial wavelength may be observed. c) Now $L = 54$, a superposition of about 20 successive states shows how the destabilization propagates. d) With $L = 54$, after a transient of about 4000 iterations (from c)) the CML relaxes to a new stable state, strongly modulated in space. e) A superposition of d) at time t and the state obtained at time $t + 2$, which shows, in fact, a period $T = 4$. f), g) A state with period $T = 2$ and wavenumber $q = 6$ is restored for $L = 48$.

Fig. 9 : Stability domains of F , ($T = 2$) for various values of λ ($WL = 2, 4, 6, 8$) in parameter space $\varepsilon \times \mu$ (ε : vertical axis, μ : horizontal axis).

Fig. 10 : Stability domain of F , ($T = 2$) for $\mu = 3.8$ in parameter space $\varepsilon \times \lambda$ (ε : horizontal axis, λ : vertical axis).

Fig. 11 : With $\mu = 3.63$, $\lambda = 8$ and $T = 4$; a) Plot of the largest value of the modulus of eigenvalues of Q_4 versus ε . b) Bifurcation diagram of a CML of length $L = 64$. For each ε 50 iterations are plotted after a transient of $t = 200$ has been died.

Fig. 12 : The same as Fig. 11-a but expressed as a function of spatial wavelength λ with $\varepsilon = 0.667$ and the same value of μ .

Fig. 13 : The same scenario as for Fig. 8.a)-g) with $\varepsilon = 0.667$, $\mu = 3.63$ and a stroboscopic period $\mathcal{T} = 4$. a) Initial state of a CML of length $L = 72$ ($q = 8$). b) The lattice just before destabilization, $L = 66$. c) Superposition of about 20 successive states when $L = 65$. d) With $L = 65$, the periodic state obtained after relaxation. e) The state of d) at time t plus the state at time $t + 4$. f) A spatiotemporal periodic state is restored when $L = 52$. ($q = 5$)

Fig. 14 : Maximal Lyapunov exponents computed along orbits close, at initial time, the cycle with $T = 2$, $\lambda = 6$, $\mu = 3.8$, versus ε : a) For reduced map F ; for each ε the orbit length is 300 after a transient of $t = 100$ has been died. b) For map \mathcal{F} with 24 Fourier modes ; for each ε the orbit length is 300 after a transient of $t = 50$ has been died.